FULL-WAVE ANALYSIS OF WAVEGUIDES INVOLVING FINITE-SIZE DIELECTRIC REGIONS

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Abstract

A moment method is presented for handling arbitrarily shaped 2D and 3D waveguides that involve conductors, finite-size dielectric regions, or both. A novel procedure for modeling the dielectric allows 2D rooftop functions to represent both the 3D polarization current in the dielectric and the surface current on the conductors. Examples include microstrip and dielectric waveguides. Numerical convergence, consistency with physical principles, and agreement with the literature are demonstrated.

INTRODUCTION

In calculating the scattering or guided-wave properties of microwave structures using moment methods, dielectric regions must often be considered. Stratified dielectrics may be accounted for by Greens functions that involve Sommerfeld integrations [1]. For dielectric regions having regular shape, modal expansions and field matching procedures may be employed [2]. Irregularly shaped regions, however, may require the use of subsectional basis function to either represent the dielectric interfaces or the dielectric volume through the polarization currents [3-5].

The volume formulations, because of limitations associated with their current expansion functions, may not be well suited for a guided-wave analysis. Pulse functions give rise to fictitious charge whose effect may not be a concern in the far field [3], but may cause serious problems in the near field. Though tetrahedral [4] or 3D rooftop functions [5] do not produce fictitious charge, a second set of basis functions would be needed to represent the surface conduction currents; and to the author's knowledge, neither basis function has been employed in a guided wave analysis. We propose a novel volume polarization formulation that offers significant advantages over the previous approaches. The versatility and accuracy of the approach are then demonstrated though the analysis of representative structures.

MODELING THE DIELECTRIC

The dielectric region is first replaced by an equivalent structure that supports surface, as opposed to volume, currents. The same surface current basis functions can therefore be used to represent both the polarization current in the dielectric and the currents on any conductors. This offers mathematical simplicity and further, allows us to make use moment method solutions already available in the literature. Second, the presence of fictitious charge can be avoided by a proper choice of basis function (rooftop function).

To represent the volume polarization with surface currents, the dielectric region is replaced by a 3D version of the thin-wall mechanism Harrington and Mautz employed to model dielectric shells [6]. As shown in Fig. 1, the dielectric is subdivided into sections along the Cartesian coordinates, so that the region is now comprised of 3D cells having dimensions τ_x, τ_y, and τ_z. If we conceptually think of the dielectric material as being pushed out from the center of each cell until it is compressed to zero thickness on the cell walls, a new structure is formed that is composed only of these zero-thickness cell walls. During compression, as the wall thickness, δ, goes to zero, the dielectric constant of the material in the wall goes to infinity as $1/\delta$. Provided the grid is sufficiently fine with respect to wavelength and to feature size, and provided that an appropriate sheet impedance is used to describe the cell walls, this new structure is electrically equivalent to the solid dielectric. (We justify this equivalence in the next section.) Since the dielectric and its cellular replacement are equivalent, we proceed to model not the solid dielectric but the equivalent structure. Because the cell walls have zero thickness, the currents that flow are precisely the 2D, surface currents that we desire.

JUSTIFICATION OF CELLULAR REPLACEMENT

The purpose of this section is convince the reader, through related examples and physical reasoning, that the cellular replacement is valid, that the accuracy of the solution technique is related to grid size, and that no formal proof is required.

The use of 2D currents to model 3D polarization currents is analogous to the modeling of a surface by a wire grid [7]. In wire-grid models, 1D basis functions are used to represent a 2D region. A serious drawback exists with wire grid modeling, however, because the field at the surface of a line current is singular. To circumvent this problem, the wires must be given an effective radius, and this may be accomplished by testing the electric field at points radially offset from their centers. Unfortunately, results are sensitive to the wire radius; fine tuning is often required to get reasonable results. Consider now a 3D grid of resistors that might be used to represent a resistive volume for the purposes of a current flow analysis.

Such a grid is actually nothing more than a wire-grid model of the volume, where the resistive material is lumped in 1D filaments that intersect at points. The value of each resistor could be taken as that across a τ_x-by-τ_y-by-τ_z cell, with the direction of current flow determining which cell dimension (τ_x,

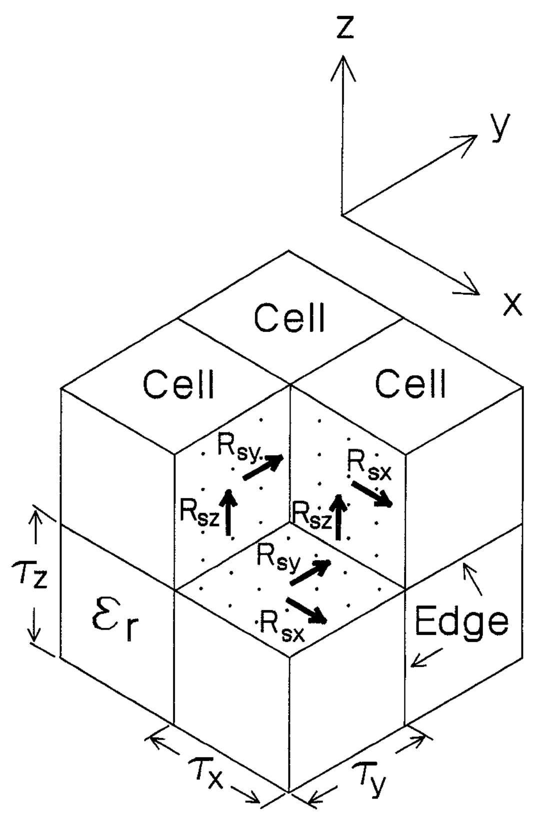

Figure 1. Dielectric subdivision, with one cell removed for illustrative purposes.

τ_y , or τ_z) serves as the length and which others form the area for a parallel plate resistance calculation. Now, if we used an array of zero-thickness rectangular sheets instead of filaments to model the volume, we would be employing the cellular replacement discussed earlier, except that it would apply to a resistive, not dielectric, volume. Each cell wall, in this case, would have to be attributed appropriate sheet resistances.

To solve electromagnetic problems with such grids, we can develop moment method solutions by applying the electric field boundary conditions; we would do this near the surface of each filament for a wire-grid model and at the surface of each cell wall for the cellular model. The use of 2D surfaces to model a volume is superior to a wire grid model, because the field of a surface current is not singular; the cellular approach does not require the fine tuning that a wire-grid model does.

Regarding the accuracy associated with the use of such grids, consider first the wire-grid resistive model. From experience, we know that for DC analyses we can achieve any desired degree of accuracy by choosing a fine enough grid. We would expect the same for the cellular model. Based on our understanding and experience with moment techniques, we expect that for an electromagnetic analysis, the wire-grid model, if properly formed, would also give improved accuracy as the grid size is reduced. We have no reason to believe that this would not be the case for the cellular model, which as described above is superior to the wire-grid model. We will further support this claim through numerical results.

In a sense, two sources of error exist in our approach; that associated with the cellular replacement and that associated with solving for the field of the equivalent structure. Both errors should approach zero as the grid size is reduced. To find the appropriate grid size, we compare numerical results against others in the literature and perform numerical convergence studies. We will show that even a relatively coarse grid gives results that range from excellent for purely dielectric structures, to well within engineering accuracy for composite structures. Many solution techniques involve grids; we just choose to apply a grid prior to the numerical solution stage.

As a further justification, consider the dielectric composition of high speed coaxial cable. Teflon is impregnated with air bubbles to reduce its effective dielectric constant. For all intents and purposes, at typical frequencies the dielectric appears homogeneous. This provides a practical example of an inhomogeneous material that is electrically equivalent to one that is homogeneous. Other examples can be found in the study of artificial dielectrics.

CHOOSING A SHEET IMPEDANCE

We chose a sheet impedance (or equivalently the surface impedance because the cell walls have zero thickness) such that the impedance is the same between two planes that sandwich a cell of either the solid dielectric or cellular replacement and include only that contribution related to the volume polarization, or in other words, we omit the free space contribution [3]. From Fig. 1, each cell wall is associated with two surface impedances. The surface impedance along x, R_{sx}, is given by

$$R_{sx} = 2\left(\frac{1}{\tau_y} + \frac{1}{\tau_z}\right) / \left[j\omega\varepsilon_0(\varepsilon_r - 1)\right] \tag{1}$$

where ω is the angular frequency, ε_0 is the permittivity of free space, ε_r is a relative dielectric constant and exp(jωt) is the time dependence. Through permutation of x,y, and z, (1) also gives the surface impedances along y and z, namely R_{sy} and R_{sz}. For walls common to two cells, the impedance would be an appropriate parallel combination. Lossy dielectrics (or for that matter, lossy conductor volumes) could be handled through appropriate choice of a complex permittivity.

The electric field boundary condition, applied over each dielectric cell wall and conductor surface, is

$$\mathbf{E} - \mathbf{J}_s R_s = 0 \tag{2}$$

where **E** is the tangential electric field, $\mathbf{J}_s$ is the surface current density, and R_s is the appropriate surface impedance. For dielectric volumes, R_s is either R_{sx}, R_{sy}, or R_{sz}; for perfect conductors, R_s is zero, but for imperfect conductors it may be determined through skin-effect considerations.

ROOFTOP REPRESENTATION AND GUIDED WAVE FORMULATION

On a dielectric or conductive surface, both full- and half-rooftops [8-10] (Fig. 2) may appear. To guarantee a smoother current distribution, we force the current to be continuous around bends by combining half-rooftops to form corner functions (Fig. 3). Internal to dielectric volumes are edges, or junctions, where three or four cell walls may intersect. At an external edge, only one corner function is needed (Fig. 3a). At three- and four-junctions, respectively, two and three corner functions are used, as shown in Fig. 3b,c. Because current flows continuously around each corner function, the total current into a junction must be zero; using more than two and three corner functions, respectively, would overspecify the current and ultimately lead to a singular matrix.

The guided wave formulation is the same one already described in the literature [8,9,11,12], except for the inclusion of R_s in equation (2). An appropriate unit cell is defined for the structure, and a moment method solution is developed that leads to a eigenvalue matrix equation of the form

$$\mathbf{Z}(k_x)\,\mathbf{I} = \mathbf{0}\,, \tag{3}$$

where k_x is the propagation constant or eigenvalue, $\mathbf{Z}$ is an impedance matrix, and $\mathbf{I}$ is a column vector of current coefficients. A Newton search is employed to find k_x, which must satisfy det($\mathbf{Z}$)=0, and then substituted back into (3) to obtain the current distribution. In the solution, the unit cell is divided in S_x, S_y, and S_z along the x, y, and z directions, respectively.

In the following examples, propagation is along the x direction. Rooftop current elements represent the currents in the x-y, y-z, and x-z planes. For the 2D structures, currents appear only on the x-z and x-y planes and only one subdivision along x is used (S_x=1); the x-directed currents will be represented only by full rooftops along the x direction.

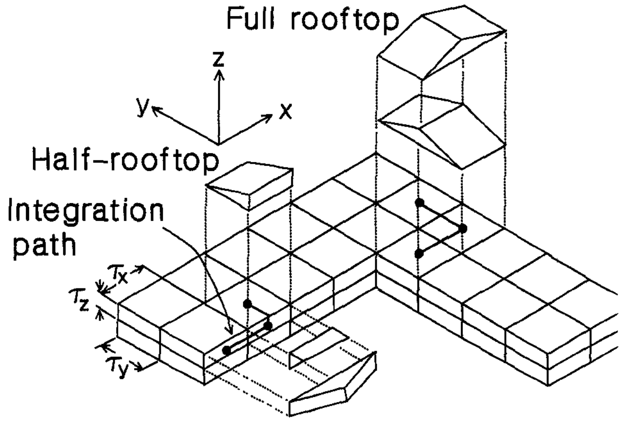

Figure 2. A section of conductor showing full and half-rooftop elements and integration paths.

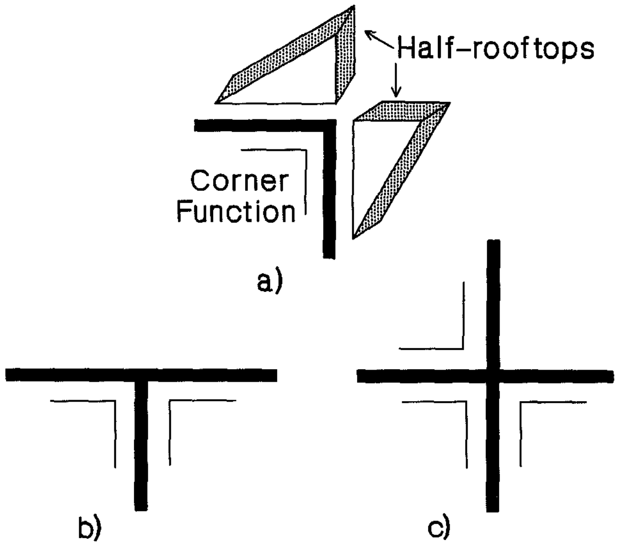

Figure 3. Representation of junction current. a) At external edge. b) At three-junction. c) At four-junction.

NUMERICAL RESULTS AND DISCUSSION

The first example is a coaxial cable filled with a dielectric material having ε_r=100, with the inside conductor having square crosssection and side b and the shield having square crosssection and side 2b. For b/λ_0=0.00067 (ie, low frequency), k_x and the per unit length capacitance C were calculated for various grid sizes. As the grid in the crosssection is made finer, from a 4-by-4 to a 32-by-32 subdivision, k_x/k_0 monotonically decreases from 10.95 to 10.04 and C decreases from 106.1 pF/cm to 91.4 pF/cm; the exact solution, based on TEM propagation and 2D capacitance calculation, is k_x/k_0=10 and C = 90.49 pF/cm. In the above, k_0 and λ_0 are the wavevector and wavelength in free space, respectively. The polarization charge in the dielectric, as expected for a homogeneous medium, is negligible.

The next structure considered is the microstrip of Fig. 4, which displays an interesting peak in effective dielectric constant ε_{eff} that is not predicted from static capacitance calculation. A 300-by-4-by-1 grid yields agreement to within four percent of that obtained using the approach of Gurel [1]. A square dielectric waveguide (Fig. 5) is then analyzed and compared with the results of Goell [2]. A similar, but hollow structure (hollow region is a/2 by a/2 and centered) is also analyzed. A periodic array of holes is then introduced into the solid waveguide, with the first stopband plotted in the figure. The grid used corresponds to an 8-by-8-by-1 breakup for the 2D problems, and an 8-by-8-by-4 breakup for the 3D problem. As expected, for wavelengths below that corresponding to the stopband (at 1.75 on the independent axis), this structure's propagation constant falls between that for the solid and hollow

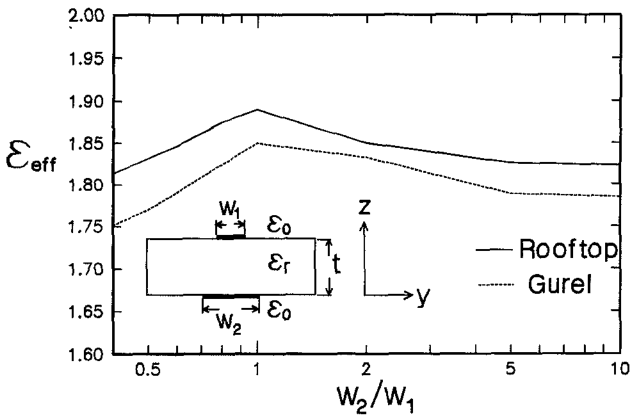

Figure 4. Effective dielectric constant in microstrip structure (w_1=0.01 cm, t=0.002 cm, ε_r=2.0, f=1.0 GHz).

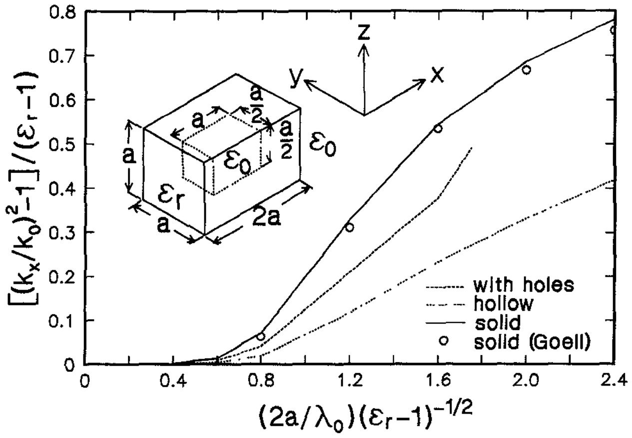

Figure 5. Propagation constant for square dielectric waveguide with and without periodic array of holes (ε_r=2.25).

cases. Near the stopband, as also expected, the curve rises dramatically.

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